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# Perturbative and nonperturbative renormalization of anomalous quark triangles

Arkady Vainshtein

*William I Theoretical Physics Institute, University of Minnesota, 116 Church St. SE, Minneapolis, MN 55455, USA*

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## Abstract

Anomalous quark triangles with one axial and two vector currents are studied in special kinematics when one of the vector currents carries a soft momentum. According to the Adler–Bardeen theorem the anomalous longitudinal part of the triangle is not renormalized in the chiral limit. We derive a new nonrenormalization theorem for the transversal part of the triangle. This nonrenormalization, in difference with the longitudinal part, holds on only perturbatively. At the nonperturbative level we use the operator product expansion and the pion dominance in the longitudinal part to determine the magnetic susceptibility of the quark condensate,  $\chi = -N_c/(4\pi^2 F_\pi^2)$ .

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## 1. Introduction

Study of fermion triangle diagrams with one axial and two vector currents represents a remarkable story. The Adler–Bell–Jackiw anomaly in the divergence of axial current, the Adler–Bardeen nonrenormalization theorem, the Wess–Zumino effective action, calculation of the  $\pi^0 \rightarrow \gamma\gamma$  amplitude, the 't Hooft consistency condition, and the solution of the U(1) problem give an incomplete list of acts where these triangles played a major role.

The famous Adler–Bardeen theorem [2] proves nonrenormalization for the longitudinal part of triangles associated with the divergence of the axial current. There is no general statement about the transversal part of the triangle. This part, and even its existence, depends on the choice of external momenta. In this Letter we argue that in special kinematics when one of the vector currents carries a soft momentum the transversal part is unambiguously fixed by the longitudinal one in the chiral limit of perturbation theory. Such relation immediately proves an absence of perturbative corrections to the transversal part of fermion triangles in the kinematics considered.

A particular physical situation where the triangle diagrams enter in special kinematics with one

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*E-mail address:* [vainshte@umn.edu](mailto:vainshte@umn.edu) (A. Vainshtein).

soft momentum occurs for the two-loop electroweak corrections to the muon anomalous magnetic moment. In Ref. [1]—the study which stimulated the present work—the nonrenormalization theorem for the transversal part is used to show an absence of gluon corrections to light quark loops.

The difference between longitudinal and transversal parts shows up at a nonperturbative level. The Operator Product Expansion (OPE) demonstrates this explicitly: in the chiral limit only the transversal part in the OPE contains nonleading operators [1]. Nonrenormalization of the longitudinal part, both perturbatively and nonperturbatively, constitutes the 't Hooft consistency condition [3], i.e., the exact quark–hadron duality. In QCD this duality is realized as a correspondence between the infrared singularity of the quark triangle and the massless pion pole in terms of hadrons.<sup>1</sup> It is clear that for the transversal part this kind of duality cannot be exact in QCD: there is no massless particle contributing to the transversal part. Thus, the transversal part of the triangle with a soft momentum in one of the vector currents provides us with an interesting object: no perturbative corrections but nonperturbatively it is modified.

An example of a nonperturbative quantity related to fermion triangles is the quark condensate magnetic susceptibility. It was introduced in Ref. [5] via the matrix element of  $\bar{q}\sigma_{\alpha\beta}q$  between the vacuum and soft photon states. Using the OPE analysis of the triangle amplitude carried on in Ref. [1] together with an additional assumption about the pion dominance in the longitudinal part we will derive a new relation for the magnetic susceptibility. This relation, similar to the Gell-Mann–Oakes–Renner relation between the pion and quark masses, is in agreement with the QCD sum rule fit [5].

## 2. Hadronic corrections to quark triangles

We follow Ref. [1] in notations and definitions. Let us start with a definition of vector,  $j_\mu$ , and axial,  $j_\nu^5$ ,

currents,

$$j_\mu = \bar{q}V\gamma_\mu q, \quad j_\nu^5 = \bar{q}A\gamma_\nu\gamma_5 q, \quad (1)$$

where the quark field  $q_f^i$  has color ( $i$ ) and flavor ( $f$ ) indices and the matrices  $V$  and  $A$  are diagonal matrices of vector and axial couplings acting on flavor indexes. To avoid dealing with the U(1) anomaly in respect to gluon interactions we assume that  $\text{Tr} A = 0$ . In the case of electroweak corrections one can view the vector current as an electromagnetic one with  $V$  being the matrix of electric charges and  $j_\nu^5$  as the axial part of the  $Z$  boson current with matrix  $A$  given by the weak isospin projection.

The amplitude for the triangle diagram in Fig. 1 involving the axial current  $j_\nu^5$  and two vector currents  $j_\mu$  and  $\tilde{j}_\gamma = \bar{q}\tilde{V}\gamma_\mu q$  (for generality we use a different matrix  $\tilde{V}$  for the soft momentum current) can be written as

$$T_{\mu\gamma\nu} = - \int d^4x d^4y e^{iqx -iky} \times \langle 0 | T \{ j_\mu(x) \tilde{j}_\gamma(y) j_\nu^5(0) \} | 0 \rangle. \quad (2)$$

We can view the current  $\tilde{j}_\gamma$  as a source of a soft photon with the momentum  $k$ . Introducing a polarization

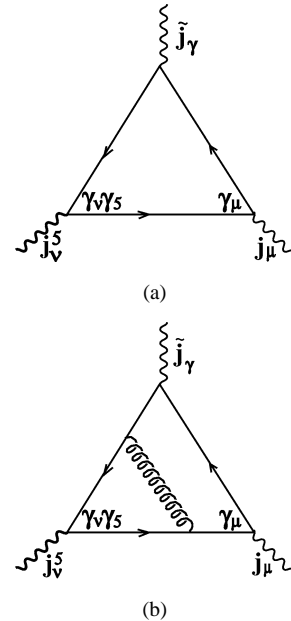


Fig. 1. Quark triangle, diagram (a), and a gluon correction to it, diagram (b).

<sup>1</sup> A pioneering effort to analyze the axial current anomaly in terms of infrared singularity was made by Dolgov and Zakharov [4].

vector of a soft photon  $e^\gamma(k)$  we come to the amplitude  $T_{\mu\nu}$

$$T_{\mu\nu} = T_{\mu\gamma\nu} e^\gamma(k) = i \int d^4x e^{iqx} \langle 0 | T \{ j_\mu(x) j_\nu^5(0) \} | \gamma(k) \rangle, \quad (3)$$

which can be viewed as a mixing between the axial and vector currents in the external electromagnetic field.

It is clear that the expansion of  $T_{\mu\nu}$  in the small momentum  $k$  starts with linear terms and we neglect quadratic and higher powers of  $k$ . There are only two Lorentz structures for  $T_{\mu\nu}$  which are linear in  $k$  and consistent with the conservation of electromagnetic current,

$$T_{\mu\nu} = -\frac{i}{4\pi^2} [w_T(q^2)(-q^2 \tilde{f}_{\mu\nu} + q_\mu q^\sigma \tilde{f}_{\sigma\nu} - q_\nu q^\sigma \tilde{f}_{\sigma\mu}) + w_L(q^2) q_\nu q^\sigma \tilde{f}_{\sigma\mu}],$$

$$\tilde{f}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\gamma\delta} f^{\gamma\delta}, \quad f_{\mu\nu} = k_\mu e_\nu - k_\nu e_\mu. \quad (4)$$

Both structures are transversal with respect to vector current,  $q^\mu T_{\mu\nu} = 0$ . As for the axial current, the first structure is transversal with respect to  $q^\nu$  while the second is longitudinal.

The one-loop result for the invariant functions  $w_T$  and  $w_L$  can be taken from the classic papers by Bell and Jackiw [6], Adler [7] and Rosenberg [8] (it simplifies considerably in the limit of the small photon momentum [9]),

$$w_L^{1\text{-loop}} = 2w_T^{1\text{-loop}} = 2N_c \text{Tr} AV \tilde{V} \int_0^1 \frac{d\alpha \alpha(1-\alpha)}{\alpha(1-\alpha)Q^2 + m^2}, \quad (5)$$

where  $Q^2 = -q^2$ , the factor  $N_c$  accounts for the color of quarks and  $m$  is the diagonal quark mass matrix,  $m = \text{diag}\{m_{q_1}, m_{q_2}, \dots\}$ . In the chiral limit,  $m = 0$ , the invariant functions  $w_{T,L}$  are

$$w_L^{1\text{-loop}}[m=0] = 2w_T^{1\text{-loop}}[m=0] = \frac{2N_c \text{Tr}(AV \tilde{V})}{Q^2}. \quad (6)$$

Nonvanishing in the chiral limit,  $m = 0$ , the longitudinal part  $q^\nu T_{\mu\nu}$  represents the axial anomaly [6,7],

$$q^\nu T_{\mu\nu} = \frac{i}{4\pi^2} Q^2 w_L q^\sigma \tilde{f}_{\sigma\mu} = \frac{i}{2\pi^2} N_c \text{Tr}(AV \tilde{V}) q^\sigma \tilde{f}_{\sigma\mu}, \quad (7)$$

and its nonrenormalization implies that the one-loop result (6) for  $w_L$  stays intact when interaction with gluons is switched on.

### 2.1. Nonrenormalization theorem for the transversal part of the triangle

We claim that the relation

$$w_L[m=0] = 2w_T[m=0], \quad (8)$$

which holds at the one-loop level, see Eq. (6), gets no perturbative corrections from gluon exchanges. This follows from the following line of argumentation.

In the chosen kinematics the fermion triangle with  $m = 0$  possesses a special feature: namely, a symmetry under permutation of indexes of axial and vector currents,  $\mu \leftrightarrow \nu$ . Indeed, in the triangle diagrams (a) and (b) in Fig. 1 one can move  $\gamma_5$  from the axial vertex  $\gamma_\nu \gamma_5$  to the vector vertex  $\gamma_\mu$ . In the chiral limit it moves via even number of gamma matrices in any order of perturbation theory. Together with the change of the momentum  $q \rightarrow -q$  (which does not affect  $T_{\mu\nu}$ ) it shows the symmetry of the amplitude  $T_{\mu\nu}$ . The same can be derived from the  $SU(N_f)_L \times SU(N_f)_R$  features of the amplitude.

At first glance the symmetry under the  $\mu \leftrightarrow \nu$  permutation seems to be in contradiction with the general decomposition (4): the transversal part of  $T_{\mu\nu}$  is antisymmetric, the longitudinal part has no symmetry, and there is no way to choose  $w_T$  and  $w_L$  which makes the  $T_{\mu\nu}$  symmetric. Note, however, that the term  $q^2 \tilde{f}_{\mu\nu}$  in the transversal structure in Eq. (4) actually produces a term in  $T_{\mu\nu}$  which does not depend on  $q$ . It is because  $w_T \propto 1/q^2$ . The  $\mu \leftrightarrow \nu$  symmetry holds for a singular in  $q$  part of  $T_{\mu\nu}$  when the condition (8) relating  $w_T$  to  $w_L$  is fulfilled. The constant in  $q$  part is then fixed by the conservation of the vector current,  $q^\mu T_{\mu\nu} = 0$ . An independence on  $q$  for this part of the amplitude provides, in fact, an alternative proof of the Adler–Bardeen theorem. Indeed, gluon corrections would lead to logarithmic dependence on  $q$  instead of the constant.

Another way to be automatically consistent with the vector current conservation is to use the Pauli–

Vilars regulators. Technically it reduces to subtraction from the triangles with massless quarks similar triangles with the heavy regulator fermions propagating on the loops. The regulator triangles produce terms which are polynomial in momenta, in our case terms linear in  $k$  and independent on  $q$ . Moreover, it is simple to see that these terms are antisymmetric under the  $\mu \leftrightarrow \nu$  permutation. Indeed, in the propagator of the heavy regulator the leading term contains no gamma-matrix that leads to the sign change when  $\gamma_5$  from the axial vertex  $\gamma_\nu \gamma_5$  is moved to the vector vertex  $\gamma_\mu$ .

Thus, we see that the crossing symmetry of the singular part in the triangle amplitude  $T_{\mu\nu}$  leads to the relation (8) in perturbation theory. Nonrenormalization of  $w_L$  implies the same for  $w_T$ .

## 2.2. Nonperturbative effects and OPE

To study a nonperturbative effect in the triangle amplitude  $T_{\mu\nu}$  one can use the OPE methods. This section is a brief review of the OPE analysis made in Ref. [1] which shows a nonperturbative origin of the difference between the longitudinal and transversal parts.

The OPE for the T-product of electromagnetic and axial currents at large Euclidean  $q^2$  has the form

$$\begin{aligned} \hat{T}_{\mu\nu} &\equiv i \int d^4x e^{iqx} T \{ j_\mu(x) j_\nu^5(0) \} \\ &= \sum_i c_{\mu\nu\gamma_1 \dots \gamma_i}^i(q) \mathcal{O}_i^{\gamma_1 \dots \gamma_i}, \end{aligned} \quad (9)$$

where the local operators  $\mathcal{O}_i^{\gamma_1 \dots \gamma_i}$  are constructed from the light fields and supplied by a normalization point  $\mu$  separating short distances (accounted in the coefficients  $c^i$ ) and large distances (in matrix elements of  $\mathcal{O}_i$ ). The field can be viewed as light if its mass is less than  $\mu$ . In the problem under consideration besides quark and gluon fields light fields include also the soft electromagnetic field  $A_\mu$  which could enter in a form of the gauge invariant field strength  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ .

The amplitude  $T_{\mu\nu}$  is given by the matrix element of the operator  $\hat{T}_{\mu\nu}$  between the photon and vacuum states,

$$\begin{aligned} T_{\mu\nu} &= \langle 0 | \hat{T}_{\mu\nu} | \gamma(k) \rangle \\ &= \sum_i c_{\mu\nu\alpha_1 \dots \alpha_i}^i(q) \langle 0 | \mathcal{O}_i^{\alpha_1 \dots \alpha_i} | \gamma(k) \rangle. \end{aligned} \quad (10)$$

In our approximation, when the matrix elements are linear in  $f_{\alpha\beta} = k_\alpha e_\beta - k_\beta e_\alpha$ , they are nonvanishing only for operators with a pair of antisymmetric indices,

$$\langle 0 | \mathcal{O}_i^{\alpha\beta} | \gamma(k) \rangle = -\frac{i}{4\pi^2} \kappa_i \tilde{f}^{\alpha\beta}, \quad (11)$$

where constants  $\kappa_i$  depend on the normalization point  $\mu$ . With only contributing operators the OPE takes the form

$$\begin{aligned} \hat{T}_{\mu\nu} &= \sum_i \{ c_T^i(q^2) (-q^2 \mathcal{O}_{\mu\nu}^i + q_\mu q^\sigma \mathcal{O}_{\sigma\nu}^i \\ &\quad - q_\nu q^\sigma \mathcal{O}_{\sigma\mu}^i) + c_L^i(q^2) q_\nu q^\sigma \mathcal{O}_{\sigma\mu}^i \}, \end{aligned} \quad (12)$$

and the invariant functions  $w_{T,L}$  can be presented as

$$w_{T,L}(q^2) = \sum_i c_{T,L}^i(q^2) \kappa_i. \quad (13)$$

The leading (by a minimal dimension) is the  $d = 2$  operator

$$\mathcal{O}_F^{\alpha\beta} = \frac{1}{4\pi^2} \tilde{F}^{\alpha\beta} = \frac{1}{4\pi^2} \epsilon^{\alpha\beta\rho\delta} \partial_\rho A_\delta, \quad (14)$$

where  $\tilde{F}^{\alpha\beta}$  is the dual of the electromagnetic field strength and the numerical factor is such that  $\kappa_F = 1$ . The OPE coefficients  $c_{L,T}^F$  for  $\mathcal{O}_F^{\alpha\beta}$  in one loop coincide with expressions (5) for  $w_{L,T}$ ,

$$\begin{aligned} c_L^F[1\text{-loop}] &= 2c_T^F[1\text{-loop}] \\ &= \frac{2N_c}{Q^2} \text{Tr} A V \tilde{V} \left[ 1 + \mathcal{O}\left(\frac{m^2}{Q^2}\right) \right], \end{aligned} \quad (15)$$

where we imply that  $m \ll \mu \ll Q$ , see [1] for a more detailed discussion.

The next, by dimension, are  $d = 3$  operators

$$\mathcal{O}_f^{\alpha\beta} = -i \bar{q}_f \sigma^{\alpha\beta} \gamma_5 q^f \equiv \frac{1}{2} \epsilon^{\alpha\beta\gamma\delta} \bar{q}_f \sigma_{\gamma\delta} q^f, \quad (16)$$

where the index denotes the quark flavor. The OPE coefficients follow from tree diagrams of the Compton scattering type,

$$c_L^f = 2c_T^f = \frac{4A_f V_f m_f}{Q^4}. \quad (17)$$

Proportionality to  $m_f$  is in correspondence with chirality arguments. Taking matrix element of  $\mathcal{O}_f^{\alpha\beta}$  between the soft photon and vacuum states we produce the following terms in the invariant functions

$w_{T,L}(q^2)$ :

$$\Delta^{(d=3)} w_L = 2\Delta^{(d=3)} w_T$$

$$= \frac{4}{Q^4} \sum_f A_f V_f m_f \kappa_f. \quad (18)$$

In perturbation theory the matrix element  $\kappa_f$  of the chirality-flip operator  $O_f$  is proportional to  $m_f$ . Nonperturbatively, however,  $\kappa_f$  does not vanish at  $m_f = 0$ . Due to spontaneous breaking of the chiral symmetry in QCD the matrix elements of quark operators (16) are instead proportional to the quark condensate  $\langle \bar{q}q \rangle_0 = -(240 \text{ MeV})^3$ . It leads to the representation of  $\kappa_f$  in the form

$$\kappa_f = -4\pi^2 \tilde{V}_f \langle \bar{q}q \rangle_0 \chi. \quad (19)$$

This representation was introduced by Ioffe and Smilga [5] in their analysis of nucleon magnetic moments and  $\Delta \rightarrow N\gamma$  radiative transitions with QCD sum rules. From a sum rule fit they determined the value of the parameter  $\chi$  dubbed as the quark condensate magnetic susceptibility,

$$\chi = -\frac{1}{(350 \pm 50 \text{ MeV})^2}. \quad (20)$$

We will consider an analytical calculation and comparison with other approaches for the susceptibility  $\chi$  in the next section. Here we notice that the effect of  $d = 3$  operators  $\mathcal{O}_f$  vanishes in the chiral limit, although as the first rather than second power of  $m$ . What we are looking for, first of all, are terms in the OPE which differentiate longitudinal and transversal parts in this limit.

Vanishing at the chiral limit persists also for the  $d = 4$  and  $d = 5$  operators. All operators of dimension 4 are reducible to the  $d = 3$  operators due to the following relation,

$$\bar{q}_f (D_\mu \gamma_\nu - D_\nu \gamma_\mu) \gamma_5 q^f = -m_f \bar{q}_f \sigma_{\mu\nu} \gamma_5 q^f. \quad (21)$$

The  $d = 5$  operators of the type  $\bar{q}_f q^f \tilde{F}^{\alpha\beta}$  and  $\bar{q}_f \gamma_5 q^f \tilde{F}^{\alpha\beta}$  enter OPE with factors  $m_f$  as in the  $d = 3$  case.

The distinction between longitudinal and transversal parts shows up at the  $d = 6$  level of four-fermion operators as it was firstly demonstrated in Ref. [10]. Referring to Ref. [1] for more detailed discussion note here that these four-fermion operators change the transversal, but not longitudinal, part. Arising due to

these operators terms  $1/Q^6$  in  $w_T$  reflect nonvanishing masses of meson resonances contributing to the transversal part. This was used in [1] for construction of a resonance model for  $w_T$  consistent with the OPE constraints.

### 3. Pion dominance and magnetic susceptibility of quark condensate

In this section we limit ourselves by the axial current of the light  $u$  and  $d$  quarks,  $j_\nu^5 = \bar{u} \gamma_\nu \gamma_5 u - \bar{d} \gamma_\nu \gamma_5 d$ , i.e.,  $A = \text{diag}\{A_u, A_d\} = \text{diag}\{1, -1\}$ .

A specific feature of the longitudinal part of  $T_{\mu\nu}$  in the chiral limit is that it is given by the leading  $d = 2$  operator  $\tilde{F}_{\alpha\beta}$  in the whole range of  $Q$ , from the ultraviolet to infrared,

$$w_L[m_{u,d} = 0] = \frac{2N_c \text{Tr}(A V \tilde{V})}{Q^2}. \quad (22)$$

At large  $Q$  it is fixed by the OPE, the pole singularity at small  $Q$  is due to massless pion with the residue that matches the OPE. How is  $w_L$  changed at nonvanishing but small  $m_{u,d}$ ?

A nonvanishing  $m_f$  implies a nonvanishing pion mass so the pole in  $w_L$  should be shifted from zero,

$$w_L[m_{u,d} \neq 0] = \frac{2N_c \text{Tr}(A V \tilde{V})}{Q^2 + m_\pi^2}. \quad (23)$$

This expression extends the pion pole dominance, which is exact at  $m_{u,d} = 0$ , to the case of small but nonvanishing  $m_{u,d}$ . Such extension is certainly valid for  $Q$  which is much smaller than the characteristic hadronic scale, say the  $\rho$  meson mass  $m_\rho$ . Moreover, at large  $Q$  the leading  $1/Q^2$  term in Eq. (23) matches what follows from the OPE. It is not enough, strictly speaking, to justify the pion dominance for the next,  $m_\pi^2/Q^4$ , term in expansion at large  $Q$ .

Assuming that the pion pole dominance for the  $1/Q^4$  term holds—we will return to this later—we can compare it with what follows from the OPE. From Eq. (23) the coefficient of  $1/Q^4$  is

$$-2m_\pi^2 N_c (V_u \tilde{V}_u - V_d \tilde{V}_d), \quad (24)$$

while the OPE relation (18) gives for this coefficient

$$\begin{aligned}
& 4(V_u m_u \kappa_u - V_d m_d \kappa_d) \\
& = 2(m_u + m_d)(V_u \kappa_u - V_d \kappa_d) \\
& \quad + 2(m_u - m_d)(V_u \kappa_u + V_d \kappa_d). \tag{25}
\end{aligned}$$

The  $m_u - m_d$  part is not relevant to comparison: it corresponds to the mixing of pion with massive isoscalar states, a reflection of the U(1) anomaly in the linear in  $m_{u,d}$  terms. Keeping in mind that  $V_{u,d}$  can be chosen arbitrarily we get

$$(m_u + m_d)\kappa_f = -m_\pi^2 N_c \tilde{V}_f, \tag{26}$$

where  $f = u, d$  and  $\tilde{V}_f$  is the electric charge of the  $f$  quark. This looks analogous to the Gell-Mann–Oakes–Renner (GMOR) relation for the pion mass [11],

$$(m_u + m_d)\langle \bar{q}q \rangle_0 = -F_\pi^2 m_\pi^2. \tag{27}$$

The GMOR relation allows us to rewrite (26) as a relation for the magnetic susceptibility  $\chi$  defined in Eq. (19),

$$\chi = -\frac{N_c}{4\pi^2 F_\pi^2} = -\frac{1}{(335 \text{ MeV})^2}. \tag{28}$$

The  $N_c$  dependence of the result for  $\chi$  is consistent with large  $N_c$  analysis. The numerical value of  $\chi$  is in very good agreement with the QCD sum rule fit [5] given in Eq. (20). What remains questionable is the pion dominance, which we will discuss in a little bit more detail.

To this end it is instructive to compare the construction above with a similar OPE derivation of the GMOR relation (27) made in Ref. [12]. The object of consideration there was the polarization operator  $\Pi_{\mu\nu}$  for the axial current  $j_\mu^5$ . In its longitudinal part the  $d = 3$  operators give

$$\begin{aligned}
\Delta \Pi_{\mu\nu}^{(d=3)} & = 2(m_u + m_d)\langle \bar{q}q \rangle_0 \frac{q_\mu q_\nu}{q^4} \\
& \quad + \text{transversal terms}. \tag{29}
\end{aligned}$$

Comparing this with the  $m_\pi^2/q^4$  term coming from the expansion in the pion pole one gets the GMOR relation (27). It is crucial that only the pion state contributes to the linear in  $m_{u,d}$  (or in  $m_\pi^2$ ) terms in the longitudinal part of  $\Pi_{\mu\nu}$ , all the higher states give quadratic in quark masses contributions. Indeed, the coupling of those states to the axial current is linear in quark masses and it is the square of this coupling which enters  $\Pi_{\mu\nu}$ . In the case of the longitudinal part

of  $T_{\mu\nu}$  the coupling of higher states to the axial current enters only once, so the higher states do contribute in the linear in quark masses order. Thus, the pion dominance is not parametrical for  $q^\nu T_{\mu\nu}$ .

A clear signal of presence of higher states follows from a nonvanishing anomalous dimension of the operator (16). It means that the operator (16) (in contrast with the operator  $(m_u + m_d)\bar{q}q$  entering the GMOR relation (27)) depends on the normalization point  $\mu$ , and its OPE coefficient  $c_L^f(Q)$  besides power dependence on  $Q$  contains also the factor  $[\alpha(Q)/\alpha(\mu)]^{16/9}$ , i.e., power of  $\log(Q/\Lambda_{\text{QCD}})$ . This logarithmic dependence is apparently related to the higher states contribution. To use the pion dominance we have to assume matching of the  $1/Q^4$  terms from the OPE and the pion pole below the higher states. It implies a low normalization point, probably  $\mu \sim 0.5 \text{ GeV}$ .

The result (28) can be compared with a different approach to calculation of  $\chi$  based on matching of the vector meson dominance with the OPE for the product of the electromagnetic current  $\tilde{j}_\gamma$  and operator (16). This approach was suggested first in Ref. [13] and in its simplest form gives  $\chi = -2/m_\rho^2 = -1/(544 \text{ MeV})^2$  what is about 2.6 times smaller by magnitude than (28). The considerations [14,15] improved by use of the QCD sum rules led to larger values, see also [16] for a recent review and update. The largest by magnitude value  $\chi[\mu = 0.5 \text{ GeV}] = -1/(420 \text{ MeV})^2$  obtained in [14] is still 1.5 times smaller than (28). A phenomenology of processes sensitive to the susceptibility  $\chi$ , see [16], will possibly help to fix its value.

## 4. Conclusions

The quark triangles in special kinematics with one soft photon are similar to polarization operators: in this case it is a nondiagonal mixing of the axial and vector currents in the background of a soft vector field. In this kinematics we find that perturbative gluon corrections are absent in the chiral limit not only for the longitudinal part but for the transversal part as well. At the nonperturbative level the transversal part is corrected in contrast with the longitudinal one. In this respect the hadronic shift in the transversal part

represents an object similar to the quark condensate: it appears only at nonperturbative level.

We also derive a new expression for the quark condensate magnetic susceptibility using the OPE and pion dominance in the longitudinal part of T-product of axial and two vector currents. This expression is similar to the Gell-Mann–Oakes–Renner relation between pion and quark masses which also can be derived by the OPE method. The crucial difference is, however, that the pion dominance is parametrically valid for the GMOR relation but not for the magnetic susceptibility. Although theoretical accuracy of the new relation is not clear it would be interesting to follow further its phenomenological consequences.

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